Temasek Defence Systems Institute

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Predicting The Accuracy of Unguided Artillery Projectiles

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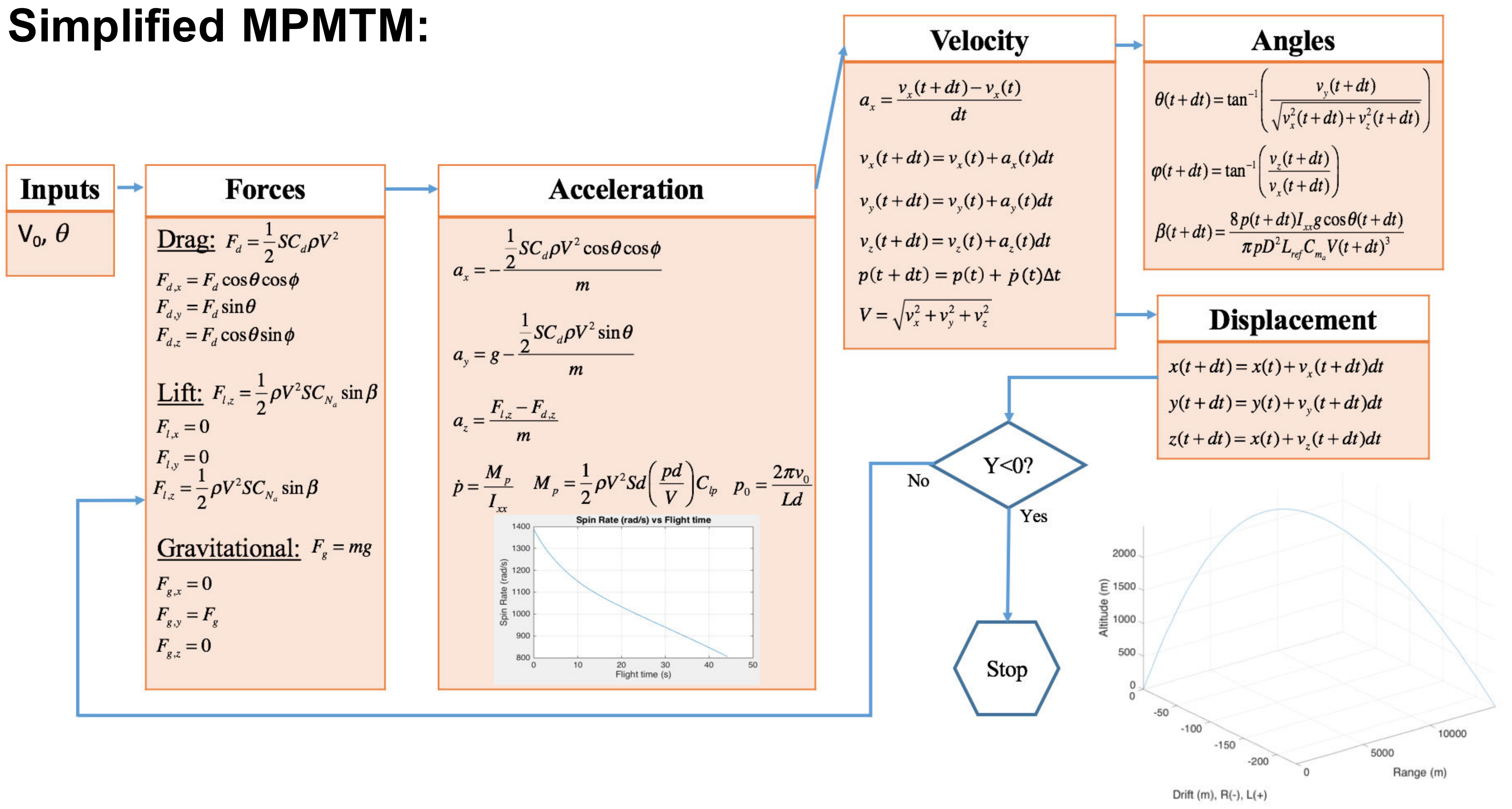
Objective: A method for predicting the accuracy of unguided artillery projectiles is presented in this thesis. The goal was to develop a standalone program that would estimate accuracy without the need for a large database of weapon trajectory data. The presented method uses a simplified version of the modified point mass trajectory model (MPMTM) and error computation models to predict error metrics that are particularly useful in predicting damage effects on various types of targets using the Joint Weaponeering System (JWS). The developed program is coded in Visual Basic, and the error metrics can typically be computed in less than 30 seconds for most ranges, in the computation precision specified in this thesis. The developed program is named the Indirect Fires Delivery Accuracy Program (IFDAP).

Simplified MPMTM:



Inputs

V_0, θ

Forces

Drag: $F_d = \frac{1}{2} S C_d \rho V^2$

$F_{d,x} = F_d \cos\theta \cos\phi$

$F_{d,y} = F_d \sin\theta$

$F_{d,z} = F_d \cos\theta \sin\phi$

Lift: $F_{l,z} = \frac{1}{2}\rho V^2 S C_{N_a} \sin\beta$

$F_{l,x} = 0$

$F_{l,y} = 0$

$F_{l,z} = \frac{1}{2}\rho V^2 S C_{N_a} \sin\beta$

Gravitational: $F_g = mg$

$F_{g,x} = 0$

$F_{g,y} = F_g$

$F_{g,z} = 0$

Acceleration

$$a_x = -\frac{\frac{1}{2} S C_d \rho V^2 \cos\theta \cos\phi}{m}$$

$$a_y = g - \frac{\frac{1}{2} S C_d \rho V^2 \sin\theta}{m}$$

$$a_z = \frac{F_{l,z} - F_{d,z}}{m}$$

$$\dot{p} = \frac{M_p}{I_{xx}} \quad M_p = \frac{1}{2}\rho V^2 S d\left(\frac{pd}{V}\right) C_{lp} \quad p_0 = \frac{2\pi v_0}{Ld}$$

Velocity

$$a_x = \frac{v_x(t+dt) - v_x(t)}{dt}$$

$$v_x(t+dt) = v_x(t) + a_x(t)dt$$

$$v_y(t+dt) = v_y(t) + a_y(t)dt$$

$$v_z(t+dt) = v_z(t) + a_z(t)dt$$

$$p(t+dt) = p(t) + \dot{p}(t)\Delta t$$

$$V = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

Angles

$$\theta(t+dt) = \tan^{-1}\left(\frac{v_y(t+dt)}{\sqrt{v_x^2(t+dt) + v_z^2(t+dt)}}\right)$$

$$\varphi(t+dt) = \tan^{-1}\left(\frac{v_z(t+dt)}{v_x(t+dt)}\right)$$

$$\beta(t+dt) = \frac{8p(t+dt) I_{xx} g \cos\theta(t+dt)}{\pi p D^2 L_{ref} C_{m_a} V(t+dt)^3}$$

Displacement

$$x(t+dt) = x(t) + v_x(t+dt)dt$$

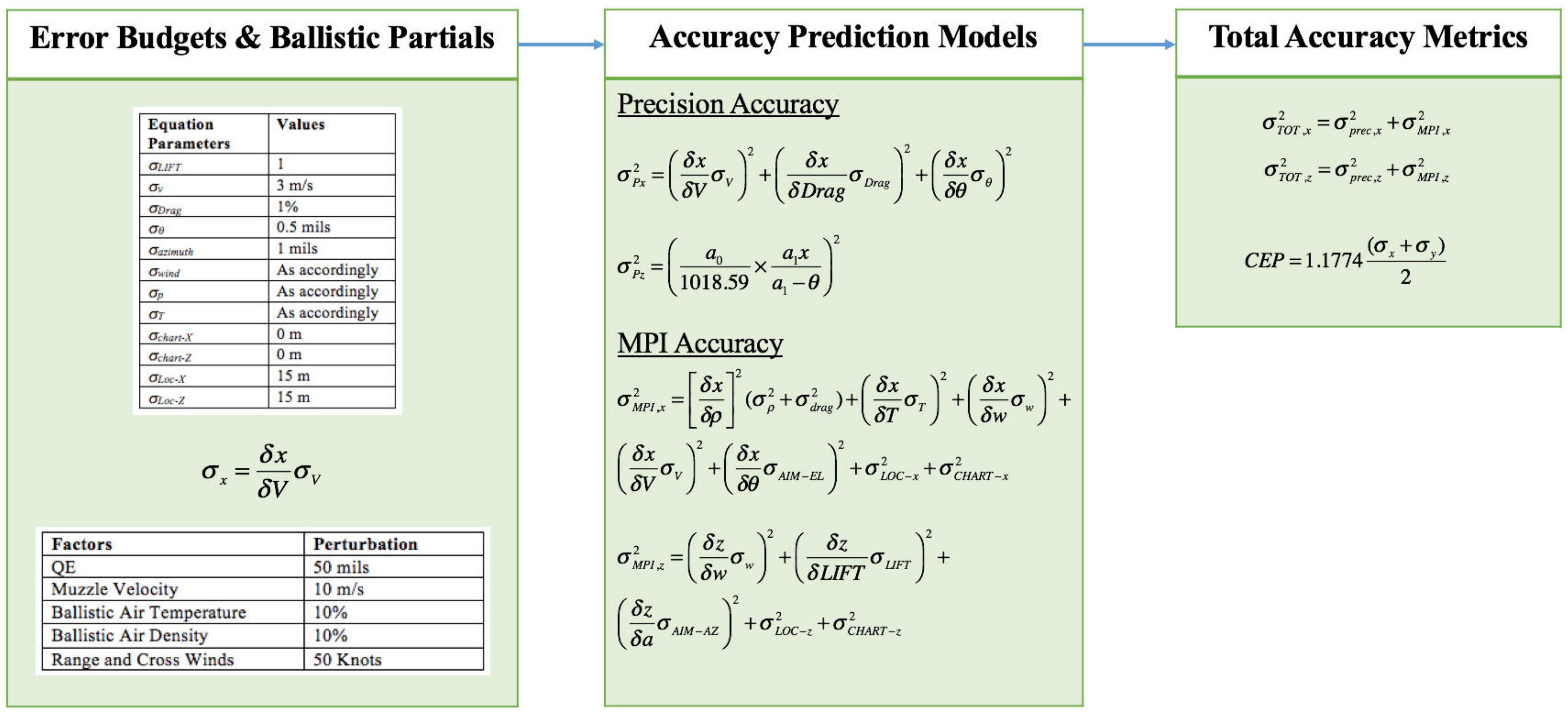
$$y(t+dt) = y(t) + v_y(t+dt)dt$$

$$z(t+dt) = x(t) + v_z(t+dt)dt$$

Y<0?

Fig 1. IFDAP Trajectory Model

Error Computations:



Error Budgets & Ballistic Partials

Equation Parameters	Values
σ_{LIFT}	1
σ_v	3 m/s
σ_{Drag}	1%
σ_θ	0.5 mils
$\sigma_{azimuth}$	1 mils
σ_{wind}	As accordingly
σ_p	As accordingly
σ_T	As accordingly
$\sigma_{chart\text{-}X}$	0 m
$\sigma_{chart\text{-}Z}$	0 m
$\sigma_{Loc\text{-}X}$	15 m
$\sigma_{Loc\text{-}Z}$	15 m

$$\sigma_x = \frac{\delta x}{\delta V}\sigma_V$$

Factors	Perturbation
QE	50 mils
Muzzle Velocity	10 m/s
Ballistic Air Temperature	10%
Ballistic Air Density	10%
Range and Cross Winds	50 Knots

Accuracy Prediction Models

Precision Accuracy

$$\sigma_{Px}^2 = \left(\frac{\delta x}{\delta V}\sigma_V\right)^2 + \left(\frac{\delta x}{\delta Drag}\sigma_{Drag}\right)^2 + \left(\frac{\delta x}{\delta\theta}\sigma_\theta\right)^2$$

$$\sigma_{Pz}^2 = \left(\frac{a_0}{1018.59} \times \frac{a_1 x}{a_1 - \theta}\right)^2$$

MPI Accuracy

$$\sigma_{MPI,x}^2 = \left[\frac{\delta x}{\delta\rho}\right]^2 (\sigma_\rho^2 + \sigma_{drag}^2) + \left(\frac{\delta x}{\delta T}\sigma_T\right)^2 + \left(\frac{\delta x}{\delta w}\sigma_w\right)^2 + \left(\frac{\delta x}{\delta V}\sigma_V\right)^2 + \left(\frac{\delta x}{\delta\theta}\sigma_{AIM-EL}\right)^2 + \sigma_{LOC-x}^2 + \sigma_{CHART-x}^2$$

$$\sigma_{MPI,z}^2 = \left(\frac{\delta z}{\delta w}\sigma_w\right)^2 + \left(\frac{\delta z}{\delta LIFT}\sigma_{LIFT}\right)^2 + \left(\frac{\delta z}{\delta a}\sigma_{AIM-AZ}\right)^2 + \sigma_{LOC-z}^2 + \sigma_{CHART-z}^2$$

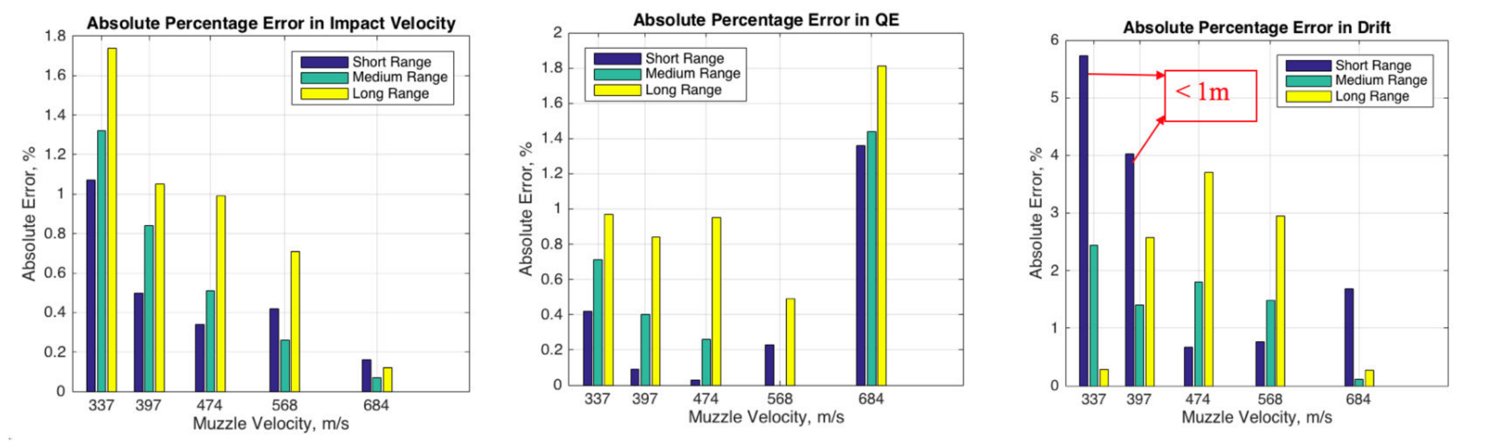
Total Accuracy Metrics

$$\sigma_{TOT,x}^2 = \sigma_{prec,x}^2 + \sigma_{MPI,x}^2$$

$$\sigma_{TOT,z}^2 = \sigma_{prec,z}^2 + \sigma_{MPI,z}^2$$

$$CEP = 1.1774\frac{(\sigma_x + \sigma_y)}{2}$$

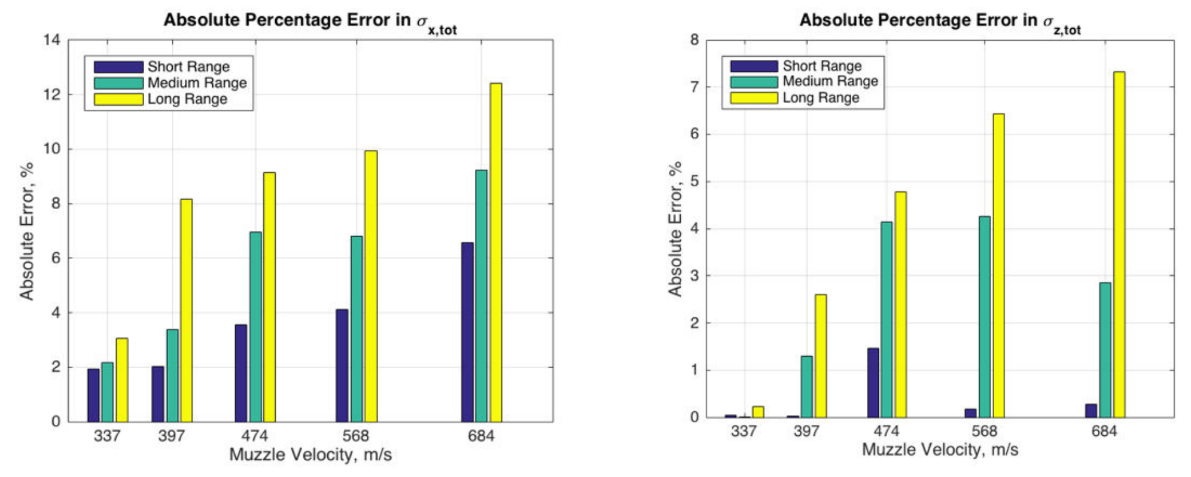
Trajectory Results Comparison with FT 155-AM-02:



The program was verified by comparing it against the FT 155-AM-02 firing table for the M107(HE) 155mm artillery projectile. The verification results demonstrate that the developed trajectory model closely matches the basic trajectory data to within 2% and ballistic partials to within 7% for most ranges of interest.

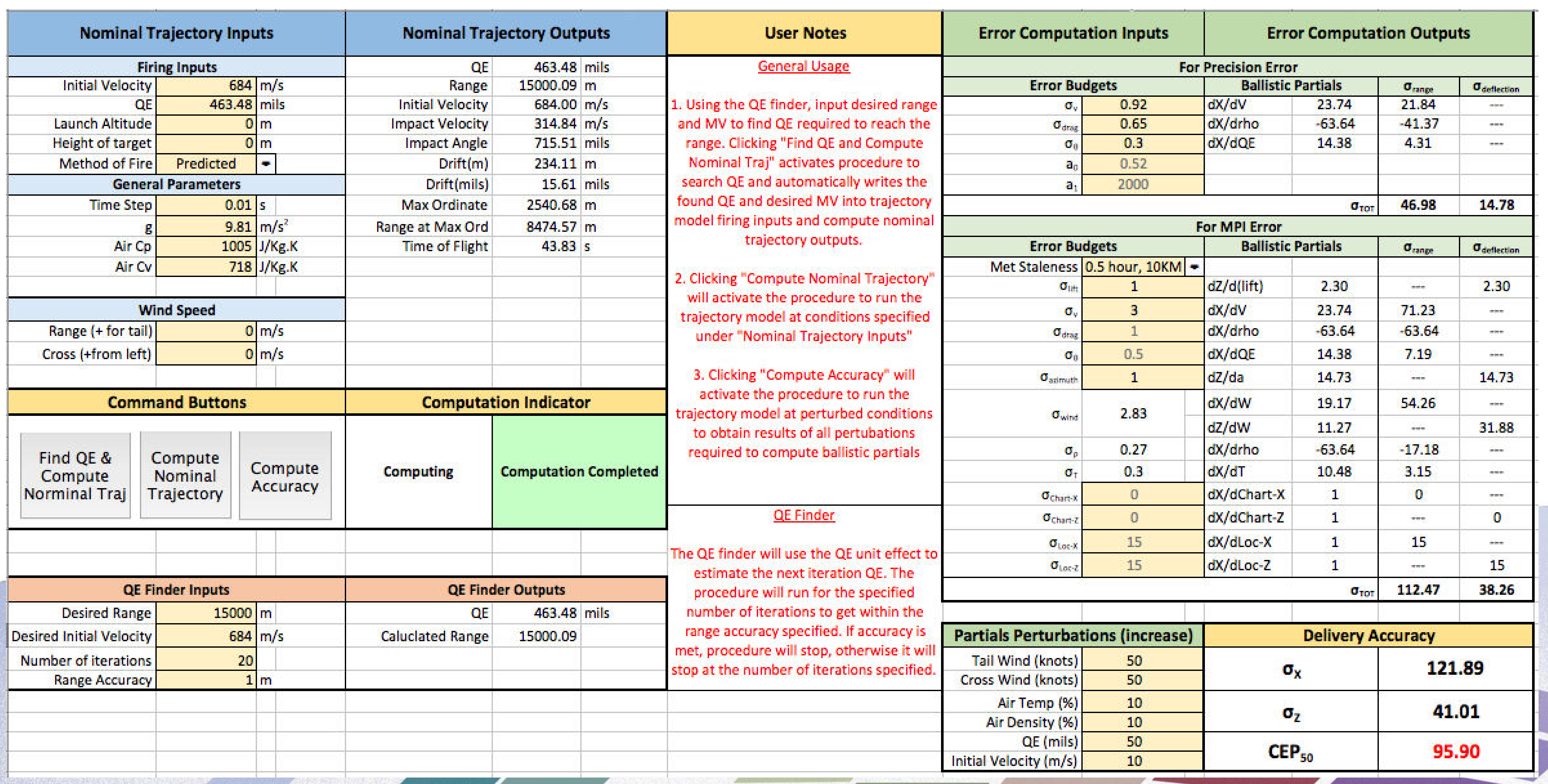
Accuracy Results Comparison with FT 155-AM-02:

Ballistic Partials	Average Error	Maximum Error
dx/dV	2.05%	4.07%
dx/dρ	4.34%	6.37%
dx/dQE	4.11%	6.77%
dx/dW	5.62%	6.94%
dz/dW	4.62%	6.94%
dx/dT	2.64%	6.00%



Accuracy metrics derived from the ballistic partials generated from the developed program are within 10% of those derived from the firing table's ballistic partials for typical firing ranges.

Main Program GUI:



Nominal Trajectory Inputs

Firing Inputs		
Initial Velocity	684	m/s
QE	463.48	mils
Launch Altitude	0	m
Height of target	0	m
Method of Fire	Predicted	
General Parameters		
Time Step	0.01	s
g	9.81	m/s²
Air Cp	1005	J/Kg.K
Air Cv	718	J/Kg.K
Wind Speed		
Range (+ for tail)	0	m/s
Cross (+from left)	0	m/s

Nominal Trajectory Outputs

QE	463.48	mils
Range	15000.09	m
Initial Velocity	684.00	m/s
Impact Velocity	314.84	m/s
Impact Angle	715.51	mils
Drift(m)	234.11	m
Drift(mils)	15.61	mils
Max Ordinate	2540.68	m
Range at Max Ord	8474.57	m
Time of Flight	43.83	s

Command Buttons

Computation Indicator

QE Finder Inputs

Desired Range	15000	m
Desired Initial Velocity	684	m/s
Number of iterations	20	
Range Accuracy	1	m

QE Finder Outputs

QE	463.48	mils
Caluclated Range	15000.09	

User Notes

General Usage

1. Using the QE finder, input desired range and MV to find QE required to reach the range. Clicking "Find QE and Compute Nominal Traj" activates procedure to search QE and automatically writes the found QE and desired MV into trajectory model firing inputs and compute nominal trajectory outputs.

2. Clicking "Compute Nominal Trajectory" will activate the procedure to run the trajectory model at conditions specified under "Nominal Trajectory Inputs"

3. Clicking "Compute Accuracy" will activate the procedure to run the trajectory model at perturbed conditions to obtain results of all pertubations required to compute ballistic partials

QE Finder

The QE finder will use the QE unit effect to estimate the next iteration QE. The procedure will run for the specified number of iterations to get within the range accuracy specified. If accuracy is met, procedure will stop, otherwise it will stop at the number of iterations specified.

Error Computation Inputs / Error Computation Outputs

For Precision Error

Error Budgets		Ballistic Partials		σ_{range}	$\sigma_{deflection}$
σ_v	0.92	dX/dV	23.74	21.84	---
σ_{drag}	0.65	dX/drho	-63.64	-41.37	---
σ_θ	0.3	dX/dQE	14.38	4.31	---
a_0	0.52				
a_1	2000				
			σ_{TOT}	46.98	14.78

For MPI Error

Error Budgets		Ballistic Partials		σ_{range}	$\sigma_{deflection}$
Met Staleness	0.5 hour, 10KM				
σ_{lift}	1	dZ/d(lift)	2.30	---	2.30
σ_v	3	dX/dV	23.74	71.23	---
σ_{drag}	1	dX/drho	-63.64	-63.64	---
σ_θ	0.5	dX/dQE	14.38	7.19	---
$\sigma_{azimuth}$	1	dZ/da	14.73	---	14.73
σ_{wind}	2.83	dX/dW	19.17	54.26	---
		dZ/dW	11.27	---	31.88
σ_ρ	0.27	dX/drho	-63.64	-17.18	---
σ_T	0.3	dX/dT	10.48	3.15	---
$\sigma_{Chart\text{-}X}$	0	dX/dChart-X	1	0	---
$\sigma_{Chart\text{-}Z}$	0	dX/dChart-Z	1	---	0
$\sigma_{Loc\text{-}X}$	15	dX/dLoc-X	1	15	---
$\sigma_{Loc\text{-}Z}$	15	dX/dLoc-Z	1	---	15
			σ_{TOT}	112.47	38.26

Partials Perturbations (increase)

Tail Wind (knots)	50
Cross Wind (knots)	50
Air Temp (%)	10
Air Density (%)	10
QE (mils)	50
Initial Velocity (m/s)	10

Delivery Accuracy

σ_X	121.89
σ_Z	41.01
CEP_{50}	95.90

Discussion & Conclusion:

1. Accuracy computation can typically be achieved in under 30 seconds for most ranges and charges.
2. The smaller the magnitude of time step, the better the convergence of the trajectory and ballistic partials outputs.
3. The angle of attack is sufficiently small to be neglected.
4. The errors are the highest at the minimum and maximum range of a given charge.

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